COST OPTIMIZATION OF SINGLY AND DOUBLY REINFORCED CONCRETET-BEAMS

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RÉSUMÉ

Cost optimization of singly and doubly reinforced concrete T-beams under ultimate design loads. An analytical approach of the problem based on a minimum cost design criterion and a reduced number of design variables, is developed. It is shown, among others, that the problem formulations can be cast intoa nonlinear mathematical programming format.Typical examples are presented to illustrate the applicability of the formulations in accordancewith the designcode BAEL91-99 currently used in Algeria. The results are then confronted to design solutions of reinforced concrete T-beams derived from current design practice. The optimal solutions show clearly that significant savings can be made in the predicted amounts and costs of the construction materials to be used.

Keywords: Non-linear Optimization, Doubly Reinforced Concrete T-beams, OptimalDesign.

ملخص

يدور هذا البحث حول حساب التكلفة المثلى للعوارض ذات مقاطعT مشكلة من الخرسانة المسلحة بصورة بسيطة ومضاعفة وفق الحالات الحدية النهائية. ولقد تم تطوير منهجية تحليلية لموضوع الدراسة على أساس تكلفة الحد الأدنى كمعيار للتصميم مع تقليل عدد متغيرات التصميم. و تبين من خلال هذه الدراسة أن المشكلة يمكن صياغتها على شكل البرمجة الرياضية الغير الخطية. و تعرض أمثلة نموذجية من أجل توضيح تطبيقات هذه المنهجية وفق القانون الفرنسي للتصميم وحساب الهياكل المشكلة من الخرسانة المسلحة BAEL91/99 والمعمول به في الجزائر. وتم إجراء مقارنة الحلول المثلى المتحصل عليها مع أساليب الحساب الكلاسيكي المتعارف عليها الخاصة بالعوارض ذات مقاطعT وقد تبن أنه تم تحقيق أرباح معتبرة في مواد البناء المستعملة.

الكلمات الرئيسية: الأمثلية الغير الخطية، التسليح المزدوج للعوارض ذات مقاطعT مشكلة من الخرسانة، التصميم المثالي.

1. INTRODUCTION

Reinforced Concrete T-beams are frequently used in industrial construction, especially in building floors, retaining walls, bridge decks, and more generally in all reinforced concrete construction projects where an appropriate portion of theslab is associated to the resisting section of the supporting beam.

For large scale use of such T-shaped beams, as may be the case for precast reinforced concrete component production, a cost effective design approach can be developed by using non-linear programming techniques. The optimal designed beams can be suitably manufactured in a prefabrication factory and then used for the specific purpose. This may lead to significant savings in the costly construction materials for both the superstructure and the foundation elements.

The total cost to be minimized is essentially divided into the construction material costs of concrete, steel and formwork. From an economical perspective, it is also desirable to consider in the design process optimization of the critical sections, the nonlinear ultimate behavior of the concrete and reinforcing steel in accordance with the currentdesign codes [1].

Within this framework the present study deals with the cost effective design of singly and doubly reinforced concrete T-shaped beams under ultimate loads. Consideration of serviceability conditions at working loadswill be addressed elsewhere as it requires further attention in terms of restrictions on bending moment capacity, stress limitation in the concrete and in steel as a function of cracking conditions, as well as limits on deflections.Such restrictions will have direct consequences on the boundaries of the design space and the feasible design solutions of the optimization problem.

The art of cost effective design consists of a procedure in which a structural optimization model is first formulated and then solved by using a suitable mathematical programming algorithm [2]. Thestructural optimization model consists of an objective function and a set of constraints.The latter generally include i) limits of search for the decision variables; ii) restrictions on structural behavior; and iii) various stress and strain conditions and their limits.Ideally, the final design must ensure to include compatibility between the geometrical dimensions of the optimized T-cross section and the ultimate loading condition including self-weight of the T-beam. Very little attention has been given to this aspect by some of the earlier investigations.

Another relevant aspect in the optimal design is the use of a particular suitable optimization algorithm. Various mathematical programming algorithms have been used in structural design optimization. This study demonstrates the formulations and the solutions of the nonlinearminimum cost design problem of singly and doubly reinforced concrete T-beams under ultimate loads by utilizing a suitable mathematical programming technique.

2. PROBLEM FORMULATION OF SINGLY REINFORCED CONCRETE T-BEAMS

Consider the T-cross section shown in Figure1 and let C0 be the objective function representing the cost of reinforced concrete and reinforcing steel to be used. This function can be defined as:

$$C_0 = L[C_c(b_0 d + (b - b_0)h_o) + C_s A_s] \qquad (1,a)$$

Where:

b:effective width of compressive flange b0:web width

h: total depth

d: effective depth

h0:flange depth

L:length of beam

As:area of reinforcingsteel

C0:absolute cost

CS:unit cost of reinforcing steel Cc:unitcost of concrete.

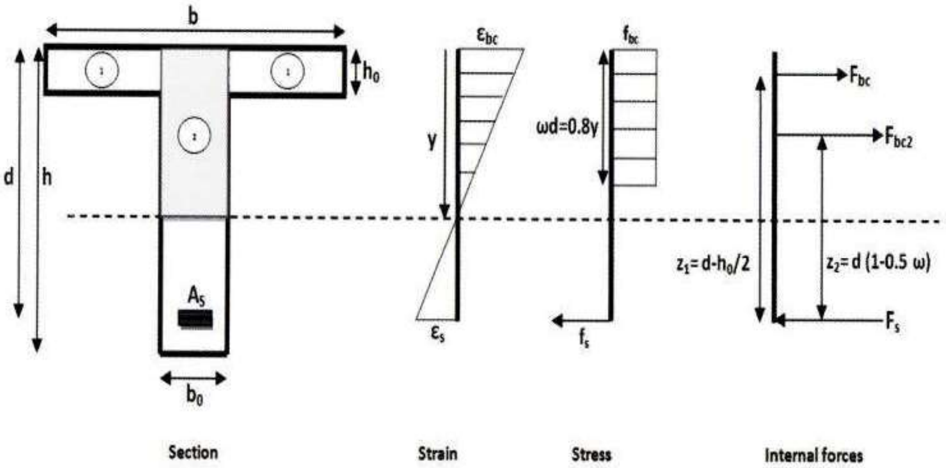


Figure1: Typical T-Cross section, strain, stress and internal force distributions under ultimate design load.

Cost components such as concrete formwork and steel forming are herein implicitly included in the objective function as appropriate percentages of the unit costs of concrete and steel respectively.

It should be equally important to note that in a cost optimization problem, the optimal values of the design variables are affected by the relative cost values of the objective function only, but not by the absolute cost values. In other words, the absolute cost values affect the final value of the objective function but not the optimal values of the design variables. Thus,the objective function to be minimized can be written as follows:

$$C = b_0 d + (b - b_0)h_o + (C_S/C_c)A_s \rightarrow Min \quad (1,b)$$

The absolute cost C0can then be recovered from the optimized relative cost C by using the relation $C_0 = C\,L\,C_c$ (1,c)

The value of the cost ratio CS / Cc varies from one country to another and may eventually depend from one region to another for certain countries.

In general, the behavior constraints are based on different design codes. For illustrative purposes, the design constraints are herein defined in accordance with the BAEL91-99 design code specifications [3], aslightly different version of the current EC2provisions. Inaddition, it is here in assumed that stress in concrete under ultimate design loads is uniformly distributed. Thus, the formulation of the minimum cost design of singly reinforced concrete T-beamsunder ultimate loads can be mathematically stated, without loss of generality, as follows:

Find the design variables b, b_0, h,d, h_0, As,and ω such that:

$$C = b_0 d + (b - b_0)h_o + \left(\frac{C_S}{C_c}\right)A_s \rightarrow Min \quad (1)$$

Subject to:

a)Behavior constraints:

$$M_u \leq f_{bc}(b - b_0)h_0(d - 0.5h_0) + f_{bc}.b_0.d^2.\omega(1 - 0.5\omega) \quad (2)$$

$$\omega = \left(\frac{f_s}{f_{bc}}\right)\left(\frac{A_s}{b_0 d}\right) - \frac{(b - b_0)h_0}{b_0 d} \quad (3)$$

$$\frac{A_s}{b_0 h + (b - b_0)h_0} \leq p_{max} \quad (4)$$

$$p_{min} \leq \frac{A_s}{b_0 d} \quad (5)$$

Conditions on strain compatibility in steel and concrete

$$\varepsilon_{els} \leq \frac{\varepsilon_{bc}(0.8 - \omega)}{\omega} \leq \varepsilon_{tsl} \quad (6)$$

$$\omega(1 - 0.5\omega) \leq \mu_c \quad (7)$$

(Compression reinforcement is not required)

b) Shear stress constraint:

$$\frac{V_u}{\bar{\tau}_u} \leq b_0 d \quad (8)$$

c)Design variables constraintsincluding rules of current practice:

$$h \geq \frac{L}{16} \quad (8)$$

$$\frac{d}{h} = 0.90 \quad (9)$$

$$0.20 \leq b_0/d \leq 0.40 \quad (10)$$

$$(b - b_0) / 2 \leq L / 10 \quad (11)$$

$$b/h_0 \leq 8 \quad (12)$$

$$h_0 \geq h_{min} \quad (13)$$

(14)

where:

C: relative cost value

M_u :maximum ultimate moment

V_u: maximum ultimate shear

ω:relative depth of compressive concrete zone

f_{c28}: compressive strength of concrete at 28 days

f_{bc}: allowable compressive stress of concrete, fbc=0.85fc28/ γ_b

γ_b: partial safety factor for concrete, γb = 1.5

f_e : elastic limit for steel

γ_s: partial safety factor for steel, γs=1.15

f_s: allowable compressive stress of steel , fed =fe/ γs

τ_u: allowable shear stress

p_{min} :minimum steel percentage

p_{max} :maximum steel percentage

E_s: modulus of elasticity of reinforcing steel

μ_c: criticalreduced moment.

μ_c = 0.392 (steel class HA FeE400)

ε_{bc}:ultimate strain of the compressive concrete, εbc=3.5‰ (in the case of Pivot B)

ε_{els}: elastic limit strain, εels =fs/Es = fe/Es.γs

ε_{els} =1.739‰ (steel HA FeE400)

ε_{tsl}: reinforcing steel tension strain limit, εtsl=10‰

The objective function equation (1) and the constraints equationsEq. (2) throughEq. (14), together form a nonlinear optimization problem. The reasons for the non-linearity of this optimization problem are essentially due to the expressions for the beam cross sectional area, the bending moment capacity and other constraints equations as well as the requirement to update iterativelythe self-weight of the element, both in the constraints functions and the objective function. Both the objective function and the constraint functions are nonlinear in terms of the designvariables.

The design variables of the model are the geometrical dimensions of the T- cross section, ratio of the amount of steel Asandω, the relative depth of the compressive concrete zone (equal to the internal axial force in steelover the total internal compressive force in the concrete).

In order to solve this nonlinear optimization problem, the GRG (Generalized Reduced Gradient) method [4]was used for the following reasons:

i)TheGRG methodis widely recognized as an efficient method for solving a relatively wide class of nonlinear optimization problems.

ii) The program can handle up to 200 constraints, which is suitable for reinforced concrete T-beam optimaldesignproblems.

iii) If necessary the program itself can estimate the Hessian matrix by using finite differences techniques.

iv) TheGeneralized Reduced Gradient (GRG) method used by the programallows for a systematic selection between the Quasi-Newton Method and the Conjugate Gradient (GG) method, depending on the availablecomputerstoragecapacity.

3. PROBLEM FORMULATION OF DOUBLY REINFORCED CONCRETE T-BEAMS

From a formulation point of view, the optimization procedure for doubly reinforced T beams in accordance with the French code BAEL91-99 provisions should proceed as described below. It should be noted that Equations 1, 2, 3 and 7 in the manuscript should be modified and equations and new constraints 19,20,24 and 28 must be added.

Consider the typical RC T-beam cross section shown in Figure2 below.

Based on the above notations, the problem formulation of doubly RC T-beams can be stated as follows:

Find the design variables b, b_0, h, d, h_0, d', A_{s1}, A_{s2} and ω such that:

$$C = b_0 d + (b - b_0)h_0 + \left(\frac{C_s}{C_c}\right)(A_{s1} + A_{s2}) \rightarrow Min \quad (15)$$

Subject to:

a) Behavior constraints:

$$M_u \le f_{bc}(b-b_0)h_0(d-0.5h_0) + f_{bc}\cdot b_0\cdot d^2.\omega(1-0.5\omega) + f_s \quad (16)$$
$$f_{bc}(b-b_0)h_0 + b_0\omega d\, f_{bc} + f_s A_{s2} - f_s A_{s1} = 0 \quad (17)$$
$$\left(M_u - \frac{(b-b_0)h_0 f_{bc}(d-0.5h_0)}{b_0}\right)/(b_0 d^2 f_{bc}) > \mu_c \quad (18)$$
$$f_s A_{s2}(d-d') \le 0.40 M_u \quad (19)$$
$$A_{s2}/(b_0 h + (b-b_0)h_0) \le p_{max} \quad (20)$$
$$A_{s1}/b_0 d \ge p_{min} \quad (21)$$
$$A_{s1}/(b_0 h + (b-b_0)h_0) \le p_{max} \quad (22)$$

Conditions on strain compatibility in steel and concrete:

$$\varepsilon_{els} \le \frac{\varepsilon_{bc}(0.8-\omega)}{\omega} \le \varepsilon_{tsl} \quad (23)$$

$$\varepsilon_{els} \le \varepsilon_{bc}\left(\omega - \frac{0.8d}{d'}\right) \le \varepsilon_{bc} \quad (24)$$

b) Shear strength constraint:

$$\frac{V_u}{\bar{\tau}_u} \le b_0 d \quad (25)$$

c) Design variables constraints including rules of current practice:

$$h \ge L/16 \quad (26)$$
$$d/h = 0.90 \quad (27)$$
$$d'/d = 0.10 \quad (28)$$
$$0.20 \le b_0/d \le 0.40 \quad (29)$$
$$(b-b_0)/2 \le L/10 \quad (30)$$
$$b/h_0 \le 8 \quad (31)$$
$$h_0 \ge h_{min} \quad (32)$$

where:

M_u : ultimate bending moment

V_u : ultimate shear

p_{min} : minimum steel percentage

p_{max} : maximum steel percentage

b : effective width of compressive flange

b_0: web width

h : total depth

h_0: flange depth

d : effective depth

d_s : effective cover of reinforcement.

h_{min}: minimum flange depth

d' : depth from the top of the compression face to the centroid of the compression reinforcement.

A_{s1} : area of tension reinforcement

A_{S2} : area of compression reinforcement

L : length of beam

f_{bc} :: Allowable compressive stress of concrete

f_s: Allowable tensile stress of steel

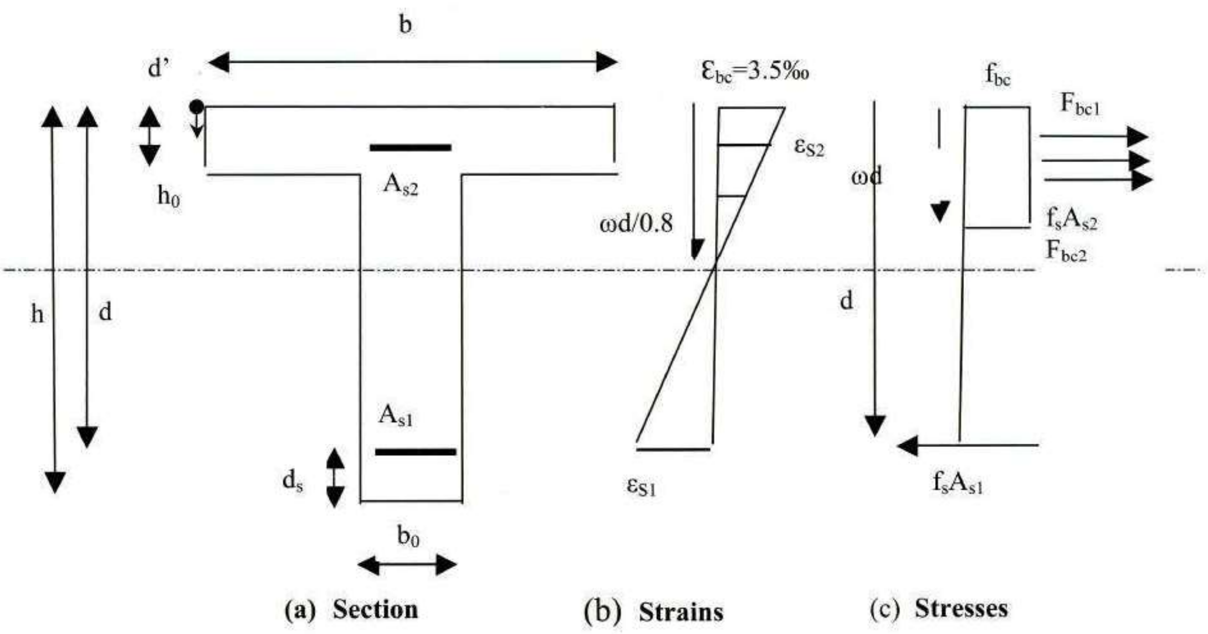


Figure 2 (a) Typical T-beam section; (b) Strains; (c) Stresses

4. NUMERICAL RESULTS AND DISCUSSION

Numerical example 1for singly RC-T-beams

In order to illustrate the applicability of the optimal design obtained by using the present formulation,the minimum cost designprocedure is now applied to design a reinforced concrete beam with T-section for which a detaileddesign solution based on the BAEL91-99 design code has been developed.

The study reinforced concrete T-beam with pined supports, along with the overhead crane (shown in most critical position so as to produce maximum bending moment under live load)is presented in Figure 3 below.

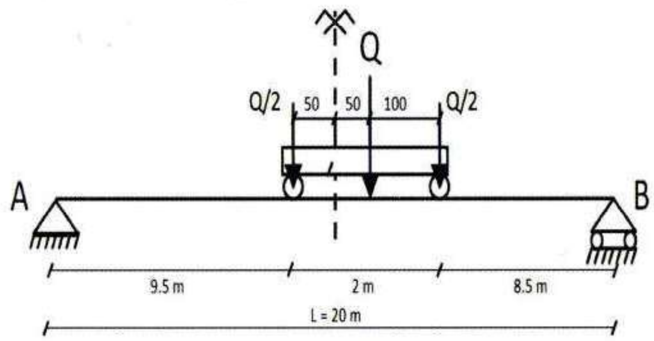


Figure 3:Overhead crane in critical position and study T-beam

The objectives of this application test are:

i)To evaluate the performance of the minimum cost design model and the solution methodology in terms of convergence to an optimal solution.

ii)To examine the characteristics of the solution in terms of the identification of binding and non- bindingconstraints.

iii)Toprovide minimum cost design solutionsof a T- beam under ultimate loads,that can be used for comparison purposes in future investigations.

As previously mentioned, the design constraints are defined in accordance with the code design specification of the French design code BAEL91-99. The corresponding pre-assigned parameters of the system are defined as follows:

-Beamspan: L= 20m

- Deadload: G= 19.5 KN/m(including initial self- weight)

- Liveload: Q= 450 KN

Combinationrule for ultimate beam capacities in bending and shear respectively

$M_u = 1.35\ MG + 1.5\ MQ$

$M_u = 4.991MNm$

$V_u = 1.35\ VG + 1.5VQ$

$V_u = 1.39MN$

Input data for concrete characteristics:

- Compressive strength of concrete at 28 days: $f_{c28} = 20MPa$

- Partial safety factor for concrete: $\gamma_b = 1.5$

- Allowablecompressive stress: $f_{bc} = (0.85fc28)/\gamma b = 11.33MPa$

-Allowable shear stress: $\tau_u = \text{Min}\ \{(0.20fc28)/\gamma_b;\ 5MPa\} = 2.67MPa$

Input data for steel characteristics:

-Elasticlimit:fe = 400MPa

- Partial safety factor for steel: γ_s =1.15
-Allowable tensile stress: $f_s = f_e/\gamma_s$ = 348MPa
-Young's elastic modulus: $E_s = 2x10^5$ MPa
-Minimum steel percentage: p_{min} = 0.1%
-Maximum steel percentage: p_{max} = 4%

Input data for unit costs of construction materials:

It should be noted that the optimal solution vector of the above nonlinear mathematical programming problem cannot be considered as the final solution of the minimum cost design problem. As a matter of fact, because of the requirement to update the geometric dimensions of the section with the new self-weight of the optimized beam, the degree of nonlinearity of the resulting optimization problem enhances further. The final optimal solution is thus obtained in two phases: Phase 1 is concerned with the determination of the optimal solution using the initial loading parameters (i.e. with initial self-weight corresponding in the startingsolution).

Phase 2is concerned with the requirement to update theself-weight of the beam (both in the constraintsfunctions and the objective function)with the geometric dimensions of the optimized sectionobtained in phase 1. The modified forces due to the new self- weight arecomputed, the newdimensions of the beam areoptimized and the process continued until convergence is achieved.

Inthe present example, the optimal solution vector is reached after 5 cycles of iteration.The vector of design variables including thegeometric dimensions of the cross section as obtained from theoptimal design solution using the proposed approach,and the classical design solution are shown in Table1.

Table 1:Final optimal solution including self-weight effects

Classical solution		Practical optimal solution	
b[m]	1.20	b[m]	1.20
b_o[m]	0.40	b_o[m]	0.30
h[m]	1.60	h_{opt}[m]	1.67
d[m]	1.46	d_{opt}[m]	1.50
h_o[m]	0.14	$h_{o\ opt}$[m]	0.15
$A_s[m^2]$	$114.61x\ 10^{-4}$	$A_{sopt}[m^2]$	$108.21x10^{-4}$
ω	0.308	ω_{opt}	0.441
C_{class}	1.108596	C_{opt}	0.97454

From the above results, it is clearly seen that the relative depth of the compressive associated with the optimal solution zone is 43% larger than that given by classical solution, thus leading to a much better use of the concrete. It is also seen that the ratio Cclass/Coptof relative costs (Cclass as obtained using BAEL classical design method to the optimal relative cost Copt)is equal to 1.14, i.e. a significant cost saving of the order of 14%as compared to the classical design solution.

The practical geometric dimensions (in cm) and the diameter of the reinforcing steel (in mm) of the optimized T-section are represented in Figure 4.

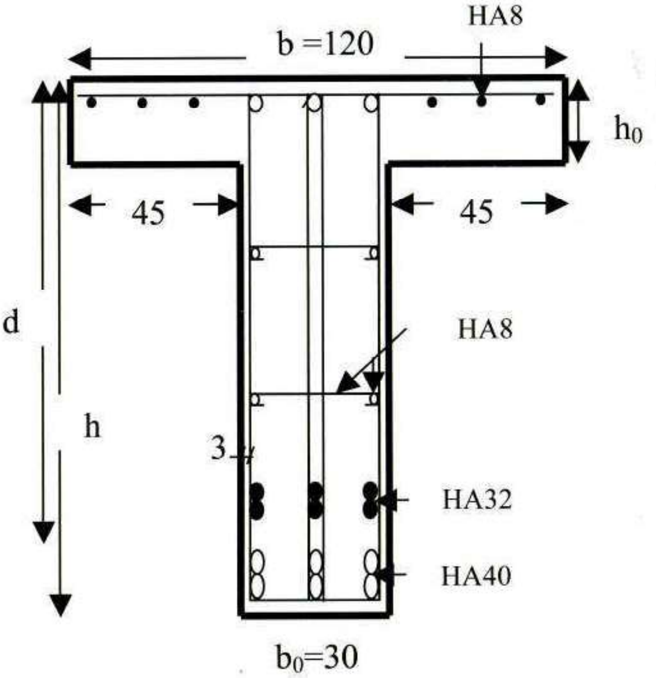


Figure 4: Final optimal solution for unit cost ratio C_s/C_c=36

A study of the constraints indicated that the design constraints of the beam are all non-bindingexcept for the behaviour constraints associated with ultimate bending moment capacity (2) and the geometrical design constraints ((10); (13); and (14)). In addition, the results obtained by using various examples have shown that the optimal solutions are insensitive to changes in the shear stress constraints whichcan thus be excluded from the problem formulation. All thickness values boand ho are on the specified lower limit values. No effort was made to explore the reason for such behaviour of the solution results.

In order to further illustrate the variability of the optimal solution with the unit cost ratio C_s/C_c, optimalsolutions have also been computed for various ratios Cs/Cc =20; 36; 50;100;150. The corresponding optimal design solution vectors are shown in Table2.In this Table, the gain in percent is defined as:

$$\text{Gain in percent (\%)} = (C_{class} - C_{opt})/C_{class} 100\%$$

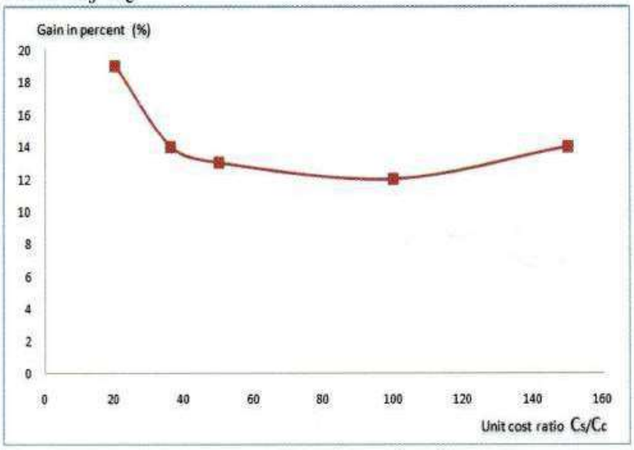
For the sake of clarity, the variation of the gain withrespect to theunit cost ratio Cs/Ccisplotted in Figure 5.

$C_s / C_c = 50$

Table 2:Optimal solutions versus unit cost ratio C_s/C_c.

C_s/C_c Optimal solution	20	36	50	100	150
b_{opt}[m]	1.26	1.18	1.20	1.20	1.20
b_{oopt}[m]	0.28	0.29	0.30	0.33	0.35
d_{opt}[m]	1.40	1.44	1.48	1.63	1.75
h_{opt}[m]	1.55	1.60	1.65	1.81	1.94
$h_{o\ opt}$[m]	0.16	0.15	0.15	0.15	0.15
A_{sopt}[m^2]	117.78×10^{-4}	115.69×10^{-4}	110.26×10^{-4}	97.97×10^{-4}	91.72×10^{-4}
ω_{opt}	0.534	0.534	0.465	0.321	0.254
C_{opt}	0.78072	0.96703	1.12546	1.64172	2.11444
C_{class}	0.92522	1.10860	1.26905	1.84210	2.41515
Gain in percent(%)	19	14	13	12	14

Figure 5:Variation of gain percentage versus unit cost ratio C_s/C_c



It can be observed that the relative gaindecreases for increasing values of the cost ratio up to the value Cs/Cc=36,and then seemsto stabilize around an average value approximately equal to 13%.

5. NUMERICAL EXAMPLE 2 FOR DOUBLY RC-T-BEAMS

A second numerical example is now devised to briefly illustrate the main results obtained for the optimal design solution using the formulation developed for doubly RC-T-beams with top reinforcement.

The corresponding pre-assigned parameters of the system are now defined as follows:

Beam span: L = 8m

Maximum design moment:

M_u = 0.980MNm

Maximum design shear: V_u = 0.490MN

Input data for concrete characteristics

Compressive strength of concrete at 28 days: f_{c28} = 25MPa

Allowable compressive stress: f_{bc} = 14.17MPa τu = 3.33MPa

Input data for steel characteristics

Steel class: HA-FeE500

Elastic limit: fe = 500MPa Allowable tensile stress: fs= 435MPa

Input data for unit costs of construction materials:

The vector of decision variables including the geometric dimensions of the cross section as obtained from the optimal design solution using the proposed approach, and the classical design solution are shown in Table 3.

Table 3: Classical and optimal solution

Classical solution		Optimal solution	
b[m]	0.90	b_{opt}[m]	0.65
b_0[m]	0.30	$b_{0\ opt}$[m]	0.19
h[m]	0.667	h_{opt}[m]	0.67
d[m]	0.60	d_{opt}[m]	0.60
h_0[m]	0.12	$h_{0\ opt}$[m]	0.12
d'[m]	0.03	d'_{opt}[m]	0.03
A_{s1}[m^2]	42.48×10^{-4}	A_{s1opt}[m^2]	41.78×10^{-4}
A_{s2}[m^2]	2.01×10^{-4}	A_{s2opt}[m^2]	10.91×10^{-4}
ω	0.290	ω_{opt}	0.362
C_{class}	0.49455	C_{opt}	0.44308

Again, it is seen thata significant cost saving ofthe order of 11% as compared to the classical design solution is obtained.

6. CONCLUSIONS

This study deals with the minimum cost design of singly and doubly reinforced concrete T-beams at ultimate design loads. An analytical approach of the problem based on a minimum cost design criterion and a set of constraints including the nonlinear behavior of concrete and steel is formulated. The set of constraints includes: i) Restrictions on the structural behavior in termsof ultimate bending moment capacityand percentage of reinforcing steel;

ii)Various compatibility conditions in terms of strains in concrete and steel, depth of neutral axis and their limits

iii) Restrictions on ultimate shear stress; and

iv) Limits of search for the decision variables

From the results of the present study, it is possible to draw the following major conclusions:

1-The problem formulations of the optimal cost design with singly and doubly of reinforced concrete T-beams can be cast into twononlinear programming problems,the numerical solution of which are efficiently determined using the GRG (Generalized Reduced Gradient) method in a space of only a few variables.

2-The space of feasible design solutions and the optimal solutions can be obtained using a reduced number of independent design variables.

3-Optimal values of the design variables are affected by the relative cost values of the objective function only, but not by the absolute cost values.

4-The observations of the optimal solutions results reveal that the use of the optimization based on the minimum-costdesign concept may lead to substantial savings in the amount of the construction materials to be used in comparison to classical design solutions of reinforced concrete T-beams.

5-The objective functions and the constraints considered in the present paper are illustrative in nature. The present approach based on nonlinear mathematical programming can be easily extended to other sections commonly used in structural design.

More sophisticatedobjectives and considerations can be readilyaccommodated by suitable modifications of the present optimalcost design model.

REFERENCES

[1] C.C.Ferreira, M.H.F.M. Barros andA.F.M. Barros, Optimal design of reinforcedconcrete T-section in bending, Engineering Structures, 25, (2003), .951-964.p

[2] A. Adamir and BL. Karihaloo, Minimum Cost Design of Reinforced Concrete T-Beams using DCOC, 2nd Computational

J.P.Topping and M. Papadrakakis (Eds), CivilComp Press, Athens, Greece, (ISBN: 0- .)1994(,)948749 International Conference on Structures Technology,B.H.V.

[3] Mougin, Béton armé BAEL91 modifié 99 et DTU associés, Eyrolles,Paris,(2006).

[4] R. Fletcher, Pratical Methods of Optimization, A Wiley-interscience Publication John Wiley & Sons, Chichester, (1987).

[5] Spon's architects and builders price book, Spon Press, London,(2011).

[6] Bordereau des prix unitaires de la construction BPUNord/Sud,EditionCNAT/CSC,Alger,(2009)